

# Math 283Z *Foundations of non-abelian Chabauty*: syllabus

Instructor: Alex Betts, abetts@math.harvard.edu

TTh 1:30–2:45pm

**Course times:** Tuesdays and Thursdays, 1:30–2:45pm, Science Center room 309a. First class on January 24th, pause for AWS and spring break 4th–18th March. There will also be supplementary office hours, Thursday 3:00–4:15pm, Science Center room 411.

**Registration:** To register, please send an email to the instructor to be added to the mailing list.

**Course description:** The non-abelian Chabauty method of Minhyong Kim is one of the premier methods for computing rational or integral points on curves in practice. This course will give a thorough description of the foundations of the method and the theoretical ingredients it combines, and will conclude with an overview of some computations of rational or integral points. The course structure is approximately as follows.

- I. **Étale fundamental groupoids.** Grothendieck’s profinite étale fundamental groupoid, pro-unipotent groupoids and pro-unipotent completion, the Tannakian formalism, the pro-unipotent étale fundamental groupoid.
- II. **Selmer schemes.** Non-abelian cohomology, pro-unipotent Kummer maps, local Selmer schemes, Selmer structures and global Selmer schemes.
- III.  **$p$ -adic fundamental groupoids.** The pro-unipotent de Rham and crystalline/rigid fundamental groupoids, Hodge filtration, the crystalline/rigid Frobenius, iterated Coleman integration, comparison isomorphisms.
- IV. **Applications and computations.** The  $S$ -unit equation, quadratic Chabauty and the Mordell–Weil sieve, survey of recent applications to modular curves.

**Prerequisites:** First courses in algebraic geometry (including over non-algebraically closed fields), algebraic topology and category theory. Some familiarity with the theory of crystalline representations and  $\varphi$ -modules would be useful but not necessary. Some background material will be covered in office hours.

**Credit:** If you wish to take this course for credit, please contact the instructor.